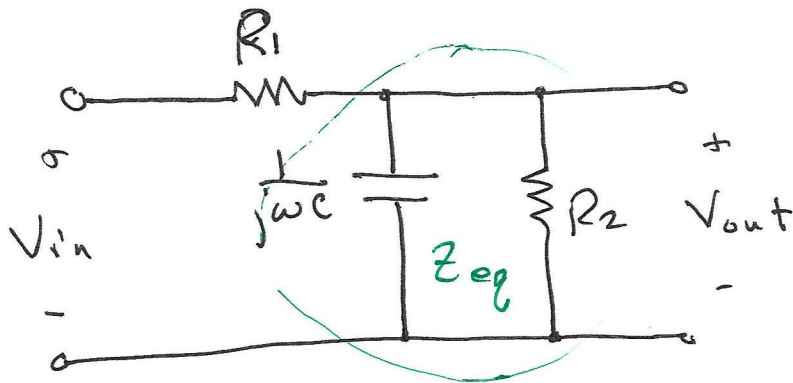


Determine the transfer function and classify in terms of frequency response.

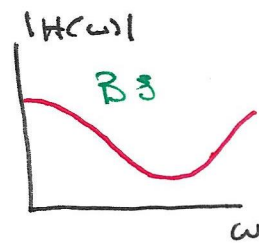
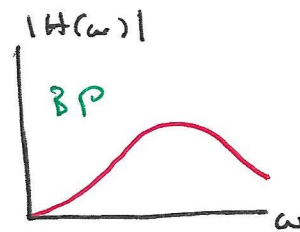
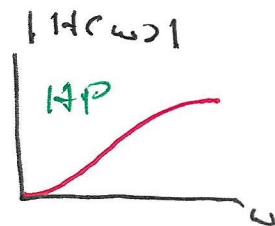
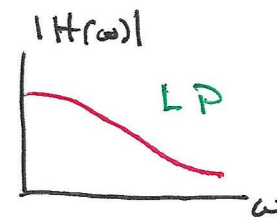
$$H(\omega) = ?$$

Method 1

Voltage divider approach.



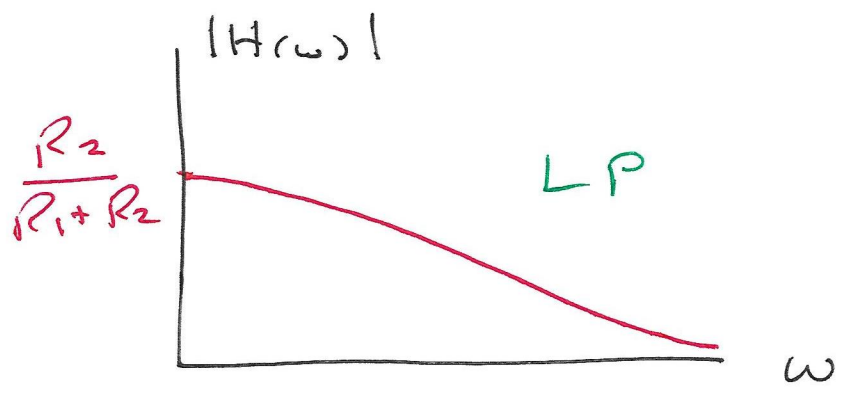
$$Z_{eq} = \frac{\left(\frac{1}{j\omega C}\right) R_2}{\frac{1}{j\omega C} + R_2} = \frac{R_2}{1 + j\omega R_2 C}$$



$$\begin{aligned}
 H(\omega) &= \frac{Z_{C_2}}{R_1 + Z_{C_2}} = \frac{\frac{R_2}{1 + j\omega R_2 C}}{R_1 + \frac{R_2}{1 + j\omega R_2 C}} = \frac{1 + j\omega R_2 C}{1 + j\omega R_2 C} \\
 &= \frac{R_2}{R_2 + R_1(1 + j\omega R_2 C)} \\
 &= \frac{R_2}{(R_1 + R_2) + j\omega R_1 R_2 C}
 \end{aligned}$$

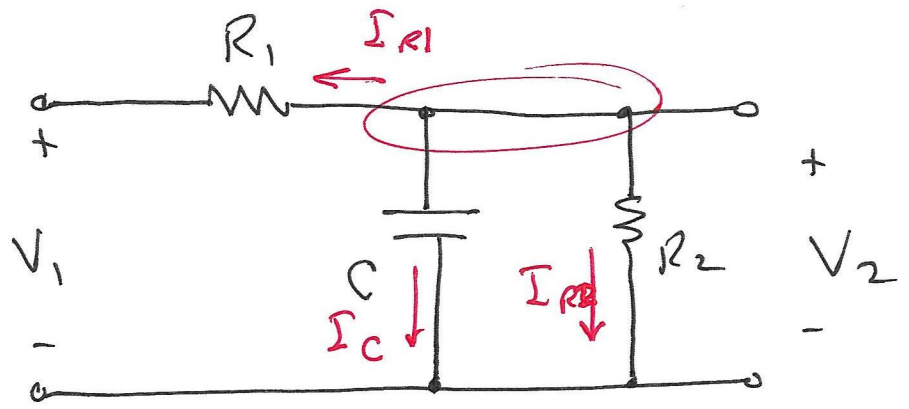
$$\lim_{\omega \rightarrow 0} |H(\omega)| = \frac{R_2}{R_1 + R_2}$$

$$\lim_{\omega \rightarrow \infty} |H(\omega)| = \left| \frac{R_2}{j\omega R_1 R_2 C} \right| = \left| \frac{1}{j\omega R_1 C} \right| = \frac{1}{\omega R_1 C} = 0$$



Method 2

KCL approach



$$I_{R1} = \frac{V_2 - V_1}{R_1}$$

$$I_C = \frac{V_2}{\left(\frac{1}{j\omega C}\right)} = j\omega C V_2$$

$$I_{R2} = \frac{V_2}{R_2}$$

KCL: $I_{R1} + I_C + I_{R2} = 0$

$$\frac{V_2 - V_1}{R_1} + j\omega C V_2 + \frac{V_2}{R_2} = 0$$

$$\left(\frac{1}{R_1} + j\omega C + \frac{1}{R_2}\right) V_2 = \frac{1}{R_1} V_1$$

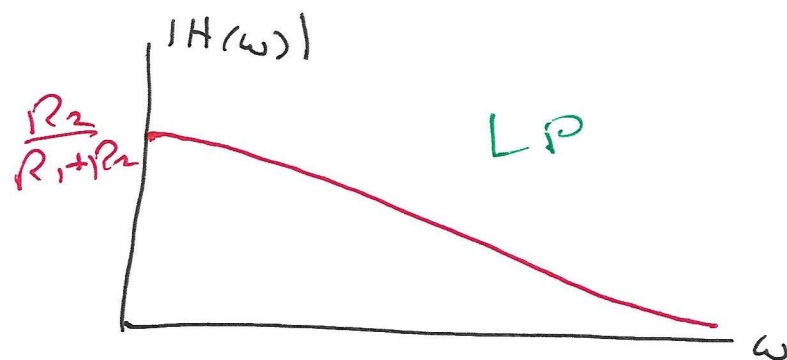
$$\frac{V_2}{V_1} = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + j\omega C + \frac{1}{R_2}} \cdot \frac{R_1 R_2}{R_1 R_2}$$

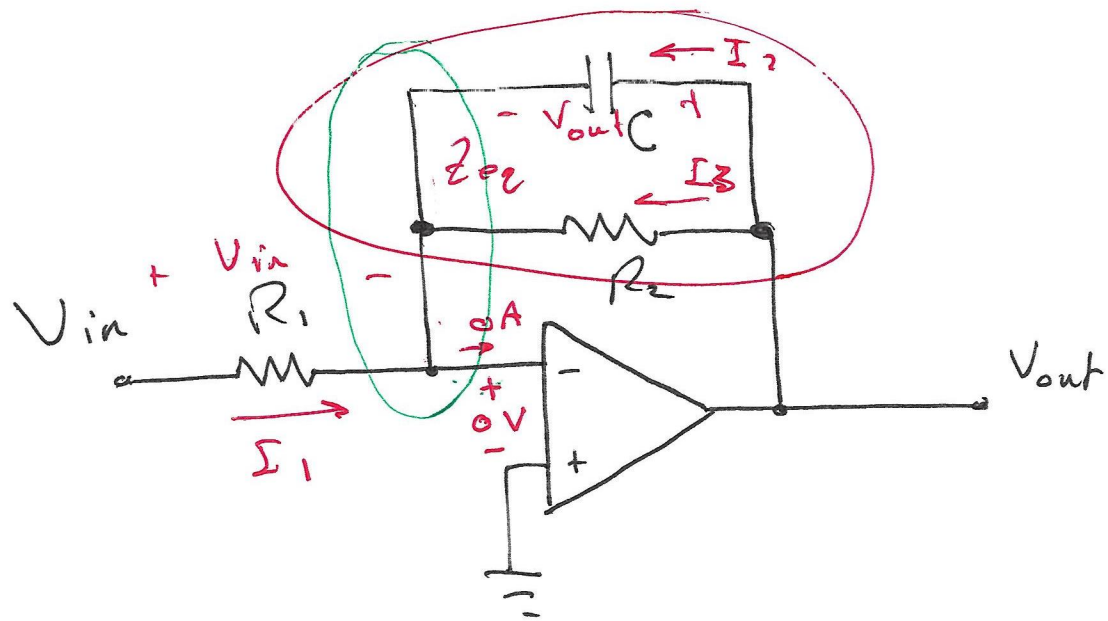
$$\frac{V_2}{V_1} = \frac{R_2}{R_2 + R_1 + j\omega R_1 R_2 C}$$

$$|H(\omega)| = \left| \frac{V_2}{V_1} \right| = \frac{R_2}{\sqrt{(R_1 + R_2)^2 + (\omega R_1 R_2 C)^2}}$$

$$\lim_{\omega \rightarrow 0} |H(\omega)| = \frac{R_2}{R_1 + R_2}$$

$$\lim_{\omega \rightarrow \infty} |H(\omega)| = \frac{R_2}{\omega R_1 R_2 C} = \frac{1}{\omega R_1 C} \rightarrow 0$$





Determine the transfer function, $H(\omega) = \frac{V_{out}}{V_{in}}$, and classify as a filter type.

Method 1

$$V_{out} = - \frac{Z_{eq}}{R_i} V_{in}$$

Method 2

$$I_1 + I_2 + I_3 = 0$$

$$I_1 = \frac{V_{in}}{R_1}$$

$$I_2 = \frac{V_{out}}{\left(\frac{1}{j\omega C}\right)} = j\omega C V_{out}$$

$$I_3 = \frac{V_{out}}{R_2}$$

$$\frac{V_{in}}{R_1} + j\omega C V_{out} + \frac{V_{out}}{R_2} = 0 \quad (\text{KCL})$$

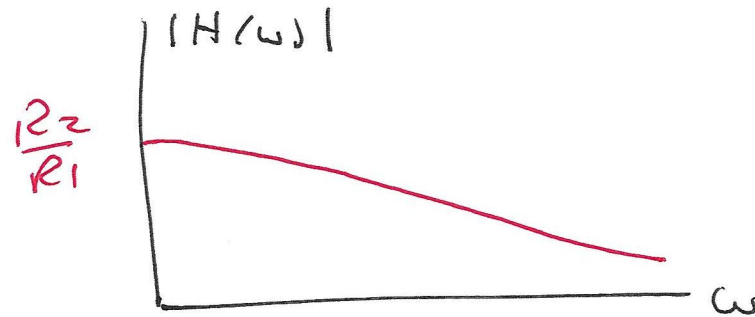
$$\left(\frac{1}{R_2} + j\omega C\right) V_{out} = -\frac{1}{R_1} V_{in}$$

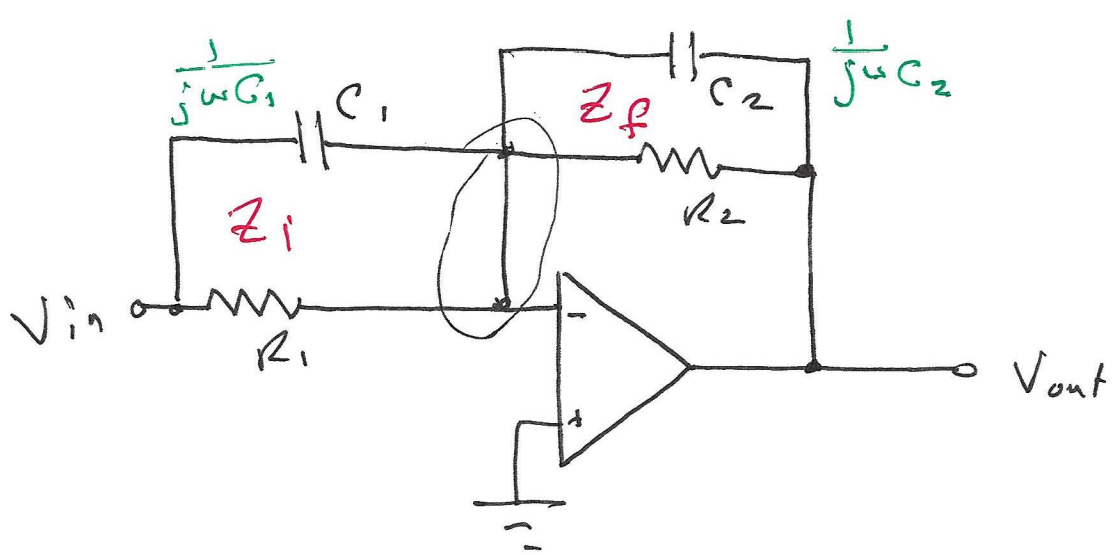
$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{-\frac{1}{R_1}}{\frac{1}{R_2} + j\omega C}$$

$$|H(\omega)| = \frac{\frac{1}{R_1}}{\frac{1}{R_2} + j\omega C}$$

$$\lim_{\omega \rightarrow 0} |H(\omega)| = \frac{\frac{1}{R_1}}{\frac{1}{R_2}} = \frac{R_2}{R_1}$$

$$\lim_{\omega \rightarrow \infty} |H(\omega)| = \frac{\frac{1}{R_1}}{j\omega C} = 0$$





Determine $H(\omega)$.

$$Z_f = \frac{R_2 \left(\frac{1}{j\omega C_2} \right)}{R_2 + \frac{1}{j\omega C_2}} = \frac{R_2}{1 + j\omega R_2 C_2}$$

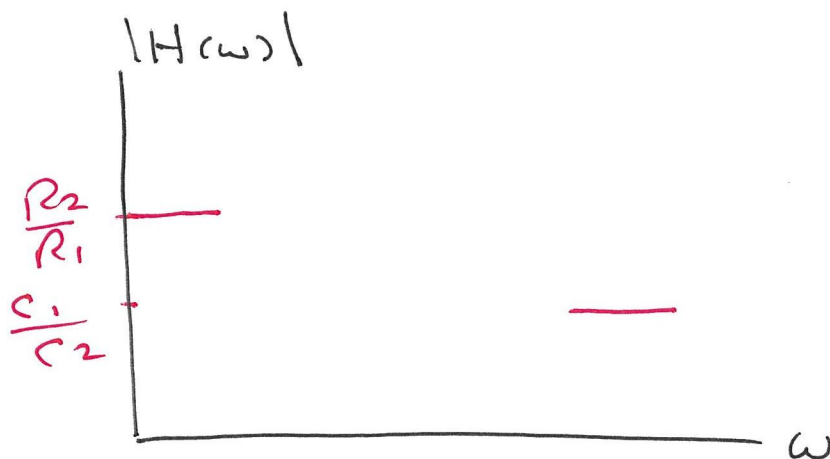
$$Z_i = \frac{R_1 \left(\frac{1}{j\omega C_1} \right)}{R_1 + \frac{1}{j\omega C_1}} = \frac{R_1}{1 + j\omega R_1 C_1}$$

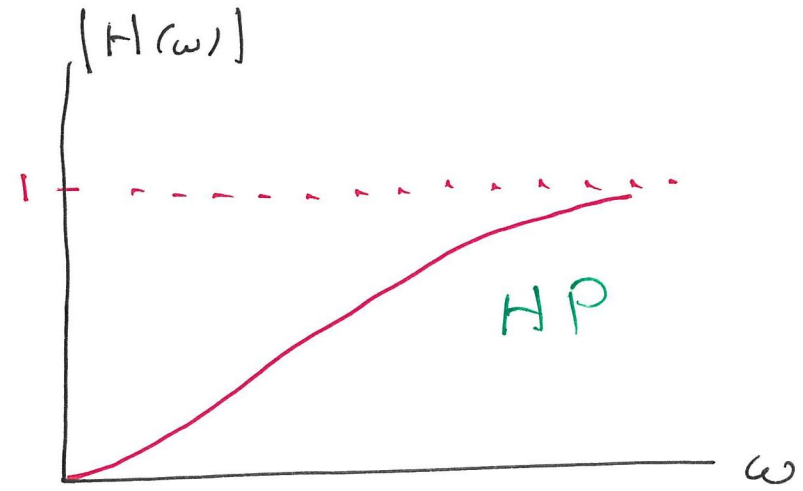
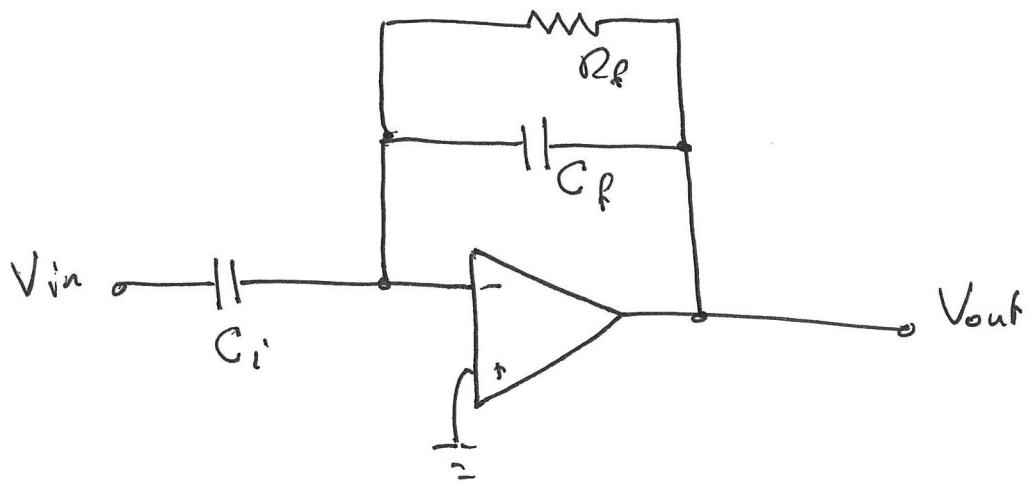
$$H(\omega) = - \frac{Z_f}{Z_i}$$

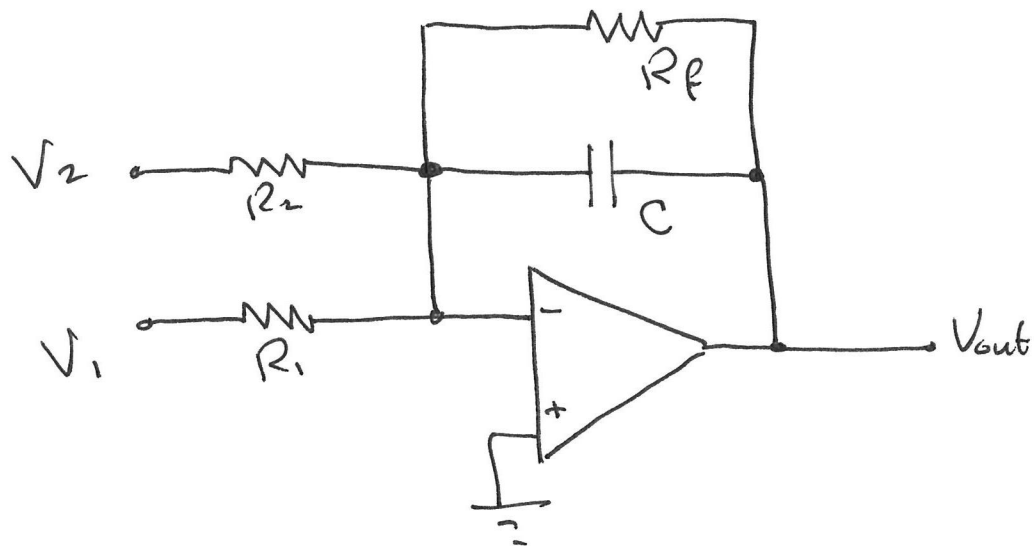
$$H(\omega) = - \frac{\left(\frac{R_2}{1+j\omega R_2 C_2} \right)}{\left(\frac{R_1}{1+j\omega R_1 C_1} \right)} = - \frac{R_2 (1+j\omega R_1 C_1)}{R_1 (1+j\omega R_2 C_2)}$$

$$\lim_{\omega \rightarrow 0} H(\omega) = - \frac{R_2}{R_1}$$

$$\lim_{\omega \rightarrow \infty} H(\omega) = - \frac{\cancel{R_2} j\omega R_1 C_1}{\cancel{R_1} j\omega R_2 C_2} = - \frac{C_1}{C_2}$$







$$V_{out} = H_1(\omega) V_1 + H_2(\omega) V_2$$

$$= - \frac{\frac{1}{R_1}}{\frac{1}{R_f} + j\omega C} V_1 - \frac{\frac{1}{R_2}}{\frac{1}{R_f} + j\omega C} V_2$$